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Inertia of a Nonradiating Particle

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Abstract

A solution of the Abraham-Lorentz equation of motion for a radiating particle is found to have nonrunaway form if its mass components are subject to nonuniform acceleration. By supposing that the energy radiated is absorbed by the particle's own field, inertia is found as a resulting property and the relation $E = Mc^2$ follows as a consequence.

Daboul (1974) has written about the persistence of the runaway solutions in the nonrelativistic Abraham-Lorentz equation of motion for a radiating particle:

$$F_{\text{ext}}(t) = m\ddot{x} - \frac{2}{3}\frac{e^2}{c^3}\dddot{x} = m(\ddot{x} - \tau\dddot{x}) \tag{1}$$

He proved that this equation and a version containing higher derivatives of $x(t)$ will always have runaway solutions if $\tau > 0$.

To avoid such runaway or preaccelerated solutions it is interesting to examine the form that the mass parameter m may take so as to transform the equation to one for which $\tau = 0$. Indeed, we will seek to give physical definition to m as that property of the particle that assures that there is no runaway and so seek to explain inertia on this basis and in terms of the electrical behavior of the particle.

When the charge e is accelerated, pulses of radiation are propagated radially at speed c. A pulse having reached radius ct has to work upon an effective mass element $\delta m(t)$ as that of the electrical field energy $\delta W(t)$ remaining to be accelerated. Thus we have

$$\delta W(t) = e^2/2(ct) \tag{2}$$

Also, since an electric field E is needed to develop the acceleration, the force on e is Ee. We will now suppose that acceleration proceeds as a discrete series of quantum events and that the whole force Ee goes to accelerate the element $\delta m(t)$ as the pulse is propagated. Thus we have

$$Ee = \delta m(t)\,\ddot{x} \tag{3}$$

Acceleration becomes a discontinuous phenomenon and varies throughout the particle field.

At a point P in the wave zone distant ct from the particle center O the electric field disturbance that gives rise to energy radiation is, according to Wilson (1946), of the form

$$e\ddot{x}\sin\theta/c^3t \tag{4}$$

where θ is the angle between OP and the direction of E or the acceleration $\ddot{x}$.

The classical energy radiation $\frac{2}{3}[e^2(\ddot{x})^2/c^3]\,\delta t$ in time δt is deduced by integrating the energy density attributable to this field term (4) for an elemental volume $2\pi(ct)^2\sin\theta\; cdtd\theta$ between limits $\theta = 0$ and $\theta = \pi$, and then doubling to allow for the equal contribution of magnetic field energy on the basis of Maxwell's equations. By introducing E, however, we find that (4) is modified to

$$e\ddot{x}\sin\theta/c^3t - E\sin\theta \tag{5}$$

and, since energy density arises from the squaring of this term, to find the time-dependent terms we are left with a quantity

$$\left(\frac{e\ddot{x}\sin\theta}{c^3t}\right)^2 - \frac{2Ee\ddot{x}}{c^3t}\sin^2\theta \tag{6}$$

This can be written as $1 - \psi(E)$ as a factor of the first term where

$$\psi(E) = \frac{2c^3tE}{e\ddot{x}} \tag{7}$$

Then equation (1) becomes

$$F_{\text{ext}}(t) = m\ddot{x} - \frac{2}{3}\frac{e^2}{c^3}\dddot{x}[1 - \psi(E)] \tag{8}$$

The condition for no runaway solution is then that $\psi(E)$ be unity. From (7) and (2) and (3), this requires a special relationship between the quantity $\delta m(t)$ and the electric field energy $\delta W(t)$. It is simply

$$\delta W(t) = \delta m(t)c^2 \tag{9}$$

We are thus led to the relationship $E = Mc^2$, where E has now its meaning as the energy attributable to the mass M. This was not implicit in the analysis, and it therefore has been derived as a consequence of our finding a solution to the Abraham-Lorentz equation.

References

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